RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

THIRD YEAR

B.A./B.SC. SIXTH SEMESTER (January – June) 2013

Mid-Semester Examination, March 2013

Date : 19/03/2013 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper - VII Full Marks : 50

[Use Separate Answer Books for each Group]

Group – A

Answer **any five** questions : [5×5]

1. Let $f:[a,b]\to\mathbb{R}$ be monotone on [a,b]. Prove that f is a function of bounded variation on [a,b]. Is the converse true? Justify. [3+2]

2. Let $f:[a,b]\to\mathbb{R}$ be a function. Prove that f is a function of bounded variation on [a,b] if and only if f can be expressed as the difference of two monotone increasing functions on [a,b]. [5]

3. Let $f(x)=|x-2|;\ x\in[0,3]$. Show that f is a function of bounded variation on [0,3]. Calculate the total variation, the positive variation and the negative variation of f on [0,3] [2+1½+1½]

4. Let $f:[a,b]\to\mathbb{R}$ be a bounded function. Prove that f is integrable on [a,b] if and only if for any $\in>0$, there exists a partition P of [a,b] such that $U(P,f)-L(P,f)<\in$ [5]

5. a) Let $f(x)=\operatorname{sgn}x, x\in[-1,3]$. Show that f is integrable on [–1,3]. Evaluate $\int_{-1}^{3} f(x)dx$. Is fundamental theorem of integral calculus applicable to evaluate $\int_{-1}^{3} f(x)dx$? Justify. [1+1+1]

   b) Let $f,g:[a,b]\to\mathbb{R}$ be both continuous on [a,b] and $\int_a^b f(x)dx=\int_a^b g(x)dx$. Prove that there exists $c\in[a,b]$ such that f(c) = g(c). [2]

6. Let $f,g:[a,b]\to\mathbb{R}$ be two bounded functions such that f(x) = g(x) except for a finite number of points in [a,b]. If f is integrable on [a,b], then prove that g is also integrable on [a,b]. [5]

7. a) State second mean value theorem of integral calculus in Weierstrass' form. Hence prove that—
   $\left|\int_a^b \frac{\sin x}{x}dx\right|<\frac{4}{a}$ where $0<a<b<\infty$. [1+2]

   b) Prove that $\frac{\pi}{6}\le\int_0^{1/2}\frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}\le\frac{\pi}{6}\frac{1}{\sqrt{1-\frac{k^2}{4}}}, k^2<1$ [2]

8. Let $f:[a,b]\to\mathbb{R}$ be integrable on [a,b] and f possess an antiderivative ϕ on [a,b]. Prove that $\int_a^b f(x)dx=\phi(b)-\phi(a)$. [5]

Group – B

Answer **any three** questions : [3×5]

9. Give the axiomatic definition of probability. Show that the conditional probabilities satisfy all the three axioms of probability. [5]

10. The probability that a man hits a target is $1/3$. How many times must he fire so that the probability of hitting the target at least once is more that 0·9? [5]

11. Find the value of the constant K so that the function f_x given by

$$f_x(x) = \begin{cases} kx(2-x), & 0 < x < 2 \\ 0 & , \text{elsewhere} \end{cases}$$

is a pdf. Construct the distribution function and compute P(x>1). [5]

12. In a textbook of 1000 pages, total number of misprints is 1000. What is the probability that a page chosen at random will contain at most 4 misprints? Also, find the probability that a page chosen at random will contain at least 4 misprints. [5]

13. The mean and variance of a binomial distribution are μ and $k\mu$ (k>0, a constant). Find the probability of no success, one success and at least one success. [5]

Group – C

Answer **any two** questions : [2×5]

14. a) Test for continuity and differentiability at $z = 0$:

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$$

b) If u(x,y) and v(x,y) are harmonic functions in a region D, prove that $u_y - v_x + i(u_x + v_y)$ is analytic in D. [3+2]

15. a) Let $f: G \to \mathbb{C}$ where $G(\subset \mathbb{C})$ is a region, be a continuous function and $g: D \to \mathbb{C}$ where D is a region & $f(G) \subset D$ and g(f(z)) = z for all z in G. If g is differentiable and $g'(z) \neq 0$, prove that f is differentiable and $f'(z) = \dfrac{1}{g'(f(z))}$.

b) Show that a real function of a complex variable either has derivative zero or the derivative does not exist. [3+2]

16. a) If $\text{Re}\{f'(z)\} = 4y - 3x^2 + 3y^2$ and f(i) = 1, find f(z).

b) Test for analyticity : $f(z) = \sin x \cosh y - i \cos x \sinh y$ [3+2]